





# CONTEST PROBLEMS







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# Problem A

### Triangles

### Time Limit: 2 seconds

There are n points, no three of which are collinear. Each point has an integer, called its beauty, representing how beautiful it is. If the integer is higher, then it means that the point is more beautiful.

The cuteness of a triangle is equal to the beauty of its least beautiful point. The hotness of a triangle is equal to the beauty of its most beautiful point. The wow factor of a triangle is equal to the product of its cuteness and hotness.

We draw all possible triangles whose vertices are three of the n points. What is the sum of the wow factors of all these triangles?

#### Input

The first line of input contains a single integer *n*.

The second line of input contains n space-separated integers  $b_1, b_2, \ldots, b_n$ .  $b_i$  denotes the beauty of the *i*th point.

#### Output

Output a single line containing a single integer denoting the answer modulo  $10^9$ .

#### Constraints

 $3 \le n \le 3 \cdot 10^5 \\ 1 \le b_i \le 10^9$ 

#### Sample Input

Sample Output

11	30904
3 9 1 2 3 9 1 100 10 55 55	







# Problem B Superconstructible

### Time Limit: 4 seconds

Not all regular *n*-gons are constructible with a compass and straightedge; Gauss was the first to discover exactly which regular *n*-gons are constructible.

If we allow for an additional tool called an *angle trisector* (which allows us trisect any given angle), then we are able to construct more regular *n*-gons. Let's call a regular *n*-gon that is constructible with a compass, straightedge, and an angle trisector **superconstructible**.

It can be shown that, for  $n \ge 3$ , the regular *n*-gon is superconstructible if and only if  $\phi(n) = 2^r 3^s$  for some nonnegative integers *r*, *s*. Here,  $\phi(n)$  denotes Euler's totient function and is defined as the number of integers in  $\{1, 2, ..., n\}$  that are relatively prime with *n*.

Given k, find the kth integer n such that  $n \ge 3$  and the regular n-gon is superconstructible.

#### Input

The first line of input contains a single integer *t* denoting the number of test cases.

Each test case consists of a single line containing a single integer k.

#### Output

For each test case, output a single line containing a single integer denoting the answer for that test case.

#### Constraints

$$1 \le t \le 150$$

 $1 \le k \le 10^{18}$ 

It is guaranteed that the answer is  $\leq 10^{18}$ 

Sample Input	Sample Output
7	3
1	4
2	5
3	6
4	7
5	240
111	13824
1206	

#### Explanation

The regular 7-gon is famously known to not be constructible with a compass and straightedge, but it is superconstructible.







On the other hand, it can be shown that a regular 11-gon is not constructible even with an angle trisector, hence it is not superconstructible.







# Problem C Jejeland

### Time Limit: 3 seconds

In Jejeland, people always write their names in whatever combination of cases they feel during that time. For example, Adonis might write his name as aDoNiS or Adonis, etc. This is a nightmare for the secretaries in Jejeland since they want to sort their files truly alphabetically. In particular, a name like Cobert will come before Cubert, irregardless of the cases of the letters. In case you have forgotten the alphabet, let us sing it together:

A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X, Y, Z. Now I know my ABCs next time won't you sing with me.

Let m and n be two names the secretary finds. Determine which one of the two names is first alphabetically. If they are the same name, then output SAME!.

#### Input

The first line of input contains a single integer *t*.

The first line of each test case contains a single string, m. The second line of each test case contains a single string, n.

#### Output

For each test case, output which of the two names is first alphabetically. If they are the same name, then output SAME!.

#### Constraints

#### 1 < t < 1000

Strings consist of English letters only.

The length of each string is between 1 and 100, inclusive.

Sample Input	Sample Output
3	abCx
abCx	ABc
CabX	SAME!
cAB	
ABc	
lol	
LOL	







# Problem D Crab Product

Time Limit: 1 second

Suppose you are given an integer  $b \ge 2$  and a non-negative integer n. We can associate a non-negative integer  $x < b^n$  uniquely to an ordered n-tuple  $(x_0, x_1, \ldots, x_{n-1})$  where  $0 \le x_i < b$  and  $x = x_0b^0 + x_1b^1 + \ldots + x_{n-1}b^{n-1}$ .

Suppose *x* and *y* are non-negative integers less than  $b^n$  where *x* is associated to  $(x_0, \ldots, x_{n-1})$  and *y* is associated to  $(y_0, \ldots, y_{n-1})$ . Then we say that their crab product is the non-negative integer *z* associated to the ordered *n*-tuple  $(z_0, \ldots, z_{n-1})$  where  $z_i$  is the largest integer satisfying  $z_i \leq x_i$  and  $z_i \leq y_i$ . We denote the crab product of two integers *x* and *y* as  $x \otimes y$ .

You are given two arrays S and T, both of length  $b^n$ . We have  $b^n$  boxes, all of which are originally empty. We index them from 0 to  $b^n - 1$ . For each ordered pair (i, j) with  $0 \le i, j < b^n$ , we put  $S_iT_j$  crab legs inside box  $i \otimes j$ . In the end, how many pieces of crab legs will each of the boxes contain? Output your answers modulo  $10^9 + 7$ .

#### Input

The first line of input contains two space-separated integers b and n.

The second line contains  $b^n$  space-separated integers  $S_0, S_1, \ldots, S_{b^n-1}$ .

The third line contains  $b^n$  space-separated integers  $T_0, T_1, \ldots, T_{b^n-1}$ .

#### Output

Output a single line containing  $b^n$  space-separated integers  $U_0, U_1, \ldots, U_{b^n-1}$ , where  $U_i$  denotes how many pieces of crab legs the *i*th box contains, modulo  $10^9 + 7$ .

#### Constraints

 $2 \le b \le b^n \le 2 \cdot 10^5$  $1 \le S_i, T_i \le 10^6$ 

Sample Input	Sample Output
3 2 3 4 50 6 2 300 3 1 30 9 7 199 5 6 6 2 4 100	6810 5066 80920 3058 3424 31980 380 224 3000







## Problem E Cookie

### Time Limit: 2 seconds

Sookie and Dookie likes to play the game Cookie. Cookie is played with n different piles of cookies, called pookies. These pookies are indexed from 1 to n, where pookie 1 is the leftmost and pookie n is the rightmost. The first m pookies have  $A_1, A_2, \ldots, A_m$  cookies. More specifically,  $A_i$  is the number of cookies on the ith pookie for  $i \leq m$ . For j > m, the jth pookie contains the same amount of cookies as the mth pookie to its left. From this description, it can be deduced how much each of the n pookies contain.

Sookie and Dookie alternate taking turns in this game. Sookie always goes first. On one turn, the player puts his hand on one of the pookies. The player then eats one or more cookies from the pookie. If the player cannot do this, then that player loses.

Before the game, Sookie goes to the toilet to make a poopie. This gives Dookie a chance to make their dog Wookie eat some of the pookies. Dookie instructs Wookie to eat a set of pookies such that:

- The pookies left are all adjacent to each other.
- There is at least one non-empty pookie left.

More formally, the cookies that should be left on the game table should all belong to pookies whose indices belong in a non-empty set of the form  $\{i, i + 1, ..., j - 1, j\}$ . How many ways can Dookie instruct Wookie to eat the pookies such that Dookie can guarantee a win, assuming both Sookie and Dookie play Cookie optimally?

#### Input

The first line of input contains a single integer *t*, the number of test cases.

The first line of each test case contains two space-separated integers *n* and *m*.

The second line contains *m* space-separated integers  $A_1, A_2, \ldots, A_m$ .

#### Output

For each test case, output a single line containing a single integer denoting the answer modulo  $10^9 + 7$ .

#### Constraints

 $1 \le t \le 5$   $1 \le m \le 10^5$   $m \le n \le 10^{18}$  $0 \le A_i \le 10^9$ 







Sample Input	Sample Output
1	6
6 4	
1 1 2 3	









# Problem F Man-in-the-Middle Attack

### Time Limit: 3 seconds

Cryptographers and security experts are very wary of different types of attacks in a cryptosystem. The most dangerous of them all is the man-in-the-middle attack. In this attack, three men arrange themselves by height. And then the man in the middle attacks.

For their safety, cryptographers follow a piece of advice from Michael Jackson that they misheard. Whenever they need to defend their cryptosystem, they're starting with the man in the middle. In particular, they want to determine which one among a group of three men will become the man in the middle.

Given the heights of three men, determine the height of the man in the middle when they arrange themselves by height. Note that if some men among the three are of the same height, these men in question arrange themselves randomly.

#### Input

The input consists of a single line containing three space-separated integers a, b, and c denoting the heights of the three men.

#### Output

Print a single line containing a single integer denoting the height of the man in the middle.

#### Constraints

 $1 \leq a, b, c \leq 1000$ 

Sample Input	Sample Output	
10 30 20	20	







# Problem G The Wedding Guests

### Time Limit: 2 seconds

There are *n* different guests invited to Marianne and Arrianne's wedding reception. We index the guests from 1 to *n*. Everyone hates each other. They intensity of how much a pair of guests hate each other is represented by an integer. Unlike love, hatred is reciprocal. If guest *i* hates guest *j* with intensity *k*, guest *j* hates guest *i* with intensity *k* as well. Weirdly enough, all guests love themselves. The intensity in which they hate themselves is 0.

In the reception, the couple expects that exactly one pair of guests will fight among each other. A pair of guests who fight in the reception will cause x thousand pesos of damage, where x is the intensity of hatred between the two fighting guests. It is best to expect the worst. In particular, one should have enough money to pay the venue if a pair who hated each other the most in the room were the pair who decided to fight. The couple prepares for this worst case and chooses to budget m thousand pesos for potential room damages, where m is the intensity of hatred between a pair of guests in the room who hate each other the most.

Marianne and Arrianne rented two rooms for the reception. Both of them know that a fight will occur in one of the rooms. They have decided that Marianne will pay for the damages of the room which will be more damaged after the reception.

The couple then strategically assigns each guest into the two rooms such that the amount that Marianne must pay in case the worst case happens is minimized. How much should Marianne expect to pay, given that they choose the optimal guest assignment?

#### Input

The first line of input contains a single integer *t* denoting the number of test cases.

The first line of each test case contains a single integer *n*.

The next *n* lines describe the intensities between the guests. Specifically, the *i*th following line contains *i* space-separated integers  $h_{i,1}, h_{i,2}, \ldots, h_{i,i}$ .

 $h_{i,j}$  represents the intensity of how much guests *i* and *j* hate each other.

#### Output

For each test case, output a single line containing a single integer denoting the answer.

#### Constraints

$$\begin{split} &1 \leq t \leq 26 \\ &1 \leq n \leq 1000 \\ &1 \leq h_{i,j} \leq 10^7 \text{ for } i \neq j \\ &h_{i,i} = 0 \end{split}$$







The sum of all  $n^2$  is  $\leq 5\cdot 10^6$ 

Sample Input	Sample Output
1	11
3	
0	
69 0	
11 42 0	









# Problem H Religious War Prevention

Time Limit: 2 seconds

A village has n different settlements and n-1 different bridges. We call the village a wellplanned village if there is a unique path (sequence of bridges) that can be taken to go from any of the n settlements to another.

There are k different groups of missionaries, each with a different religion. They can influence settlements to become their religion. Once a settlement accepts their religion, they will be of that religion forever. At the end of the missions of the k different groups, each settlement has accepted a religion. A group of missionaries may have influenced as many as all n settlements or as few as none at all.

One of the teachings of all k religions is to make sure that if two settlements are of the same religion, then all the settlements in the unique path between them should be of the same religion as they are. Otherwise, the god of their religion will punish them. This usually results in a war between settlements. We don't want that.

Thus, the k groups of missionaries only wish to influence well-planned villages with exactly n elements. Given that they are all peace-loving religions, the groups decide amongst themselves that they will influence settlements of these well-planned villages such that no war will result based on the way they influenced them. Let  $w_V$  be the number of ways they can do this for a certain well-planned village V. Among all possible well-planned villages V, let mbe the minimum and let M be the maximum value of  $w_V$ . Find m and M if you are given nand k.

#### Input

The first line of input contains t, the number of test cases. Each test case consists of a single line containing two space-separated integers n and k.

#### Output

For each test case, output a single line containing two space-separated integers *m* and *M*, both modulo  $10^9 + 7$ .

#### Constraints

 $\begin{array}{l} 1 \leq t \leq 1500 \\ 1 \leq n, k, nk \leq 10^9 \end{array}$ 

Sample Input Sample Output		
1	6 6	
3 2		







# Problem I The Pangets

### Time Limit: 2 seconds

There is a group of people, collectively known as The Pangets, who have decided to live in their own set of islands. These islands are indexed from 1 to n. The *i*th island has been blessed  $g_i$  times.

The Pangets have decided to live amongst themselves, away from the real world, because their girlfriends lie about them. In particular, whenever a friend asks their girlfriend what type of person they are, the girlfriends respond by saying "Mabait" or "Maginoo" instead of the truth. The Pangets pride themselves of being panget and so such answers from their significant others cause great offense.

The amount of panget one Panget has is quantified by their Pangetness. Pangetness is a non-negative integer r.

There are n islands in the Panget Archipelago. There are also b different bridges. Each bridge connects a pair of distinct islands.

Each bridge has a Panget Requirement. The Panget Requirement is a non-negative integer *r*. A Panget can only cross a bridge if their Pangetness is not less than the Panget Requirement of that specific bridge.

The Panget Island has an annual tradition lasting q days. For each day, there will be two special islands s and t. Let m be the minimum Pangetness required to be able to go from island s to island t using only the b bridges. Exactly one Panget Priest from island s with exactly m Pangetness is chosen. He blesses each island he can reach from island s exactly once before the sun sets. This means that he visits all islands that he can at least once during the day, and for every island i that he reaches,  $g_i$  is increased by one. After the sun sets, find the sum of the  $g_i$  over all islands visited by that day's Panget Priest.

#### Input

The first line of input contains three space-separated integers *n*, *b* and *q*.

The second line contains *n* space-separated integers  $g_1, g_2, \ldots, g_n$ .

The next b lines describe the bridges. Specifically, each of the next b lines contains three space-separated integers x, y, r which means that a bridge that has a Panget Requiremen of r connects islands x and y.

The next q lines describe the days of the annual tradition. Specifically, each of the next q lines contains two space-separated integers s and t.

#### Output

For each day of the annual tradition, print the sum of the  $g_i$  over all islands visited by that day's Panget Priest.







#### Constraints

 $2 \le n \le 150000$   $1 \le b \le 200000$   $1 \le q \le 100000$   $1 \le r, g_i \le 10^6$   $1 \le x, y, s, t \le n$   $x \ne y$  $s \ne t$ 

There is a path between any pair of islands.

Sample Input	Sample Output
579	508
69 6 9 69 420	512
1 2 11	586
2 3 10	591
5 4 25	25
1 5 60	598
3 4 20	603
2 4 50	534
2 5 11	538
1 2	
1 5	
1 4	
4 5	
2 3	
2 4	
3 4	
1 2	
1 2	







# Problem J Fififibobobobonacci Sequence

Time Limit: 2 seconds

The  $(s_0, s_1)$ -<u>fi $\cdots$ fi bo $\cdots$ bo</u> nacci sequence is a sequence whose 0th term is  $s_0$ , 1st term is

 $s_1$ , and whose *i*th term is  $as_{i-1} + bs_{i-2}$  when  $i \ge 2$ . For example, the first few terms of the (1,2)-fififibobobobonacci sequence starts with 1,2,10,38,154,...

Jingdong Dantes makes a  $u \times v$  table. He writes the terms of the  $(s_0, s_1)$ -fi $\cdots$  fi bo  $\cdots$  bo nacci

sequence in his table, starting on the cell on the first row and first column of the table. He writes the rest of the sequence going left to right. If he reaches the end of a row, he starts on the first cell of the next row and so on.

Similarly, Jengdung Dantes makes a  $u \times w$  table. Just like his brother Jingdong, Jengdung writes the terms of the  $(s_0, s_1)$ -fi $\cdots$  fibo $\cdots$  bo nacci sequence in his table, starting on the

cell on the first row and first column of the table. He writes the rest of the sequence going left to right. If he reaches the end of a row, he starts on the first cell of the next row and so on.

Jingdong chooses one of the columns in his table as his favorite, say column *m*. Jengdung chooses his favorite column from his table as well, say column *n*. Let  $t_j$  be the product of the *j*th number on Jingdong's favorite column with the *j*th number on Jengdung's favorite column. Find the sum  $t_1 + \ldots + t_u$ .

Since the sum can be very large, output it modulo  $10^9 + 7$ .

*Note:* An  $r \times c$  table is a table with r rows and c columns.

#### Input

The first line contains a single integer *t* denoting the number of test cases.

Each test case consists of one line containing nine integers  $a, b, m, n, u, v, w, s_0, s_1$ .

#### Output

Output a single line containing a single integer denoting the answer.

#### Constraints

 $1 \le t \le 8000 \\ 1 \le a, b, u, v, w, s_0, s_1 \le 10^{18} \\ 1 \le m \le v \text{ and } 1 \le n \le w$ 







Sample Input	Sample Output
8	1579266
1 1 2 5 3 6 7 1 1	273843983
3 4 2 5 3 6 7 1 2	374629028
3 4 2 5 300 6 7 1 2	899749965
3 4 2 5 30000 6 7 1 2	876364534
3 4 2 5 3 6 7 6 9	113653566
3 4 2 5 300 6 7 6 9	159483974
3 4 2 5 30000 6 7 6 9	816278509
420 69 6 9 420 69 96 1234 4321	

#### Explanation

The first test case corresponds to the traditional (1, 1)-Fibonacci sequence:

 $1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, \ldots$ 

Jingdong Dantes makes a  $3 \times 6$  table:

1	1	2	3	5	8
13	21	34	55	89	144
233	377	610	987	1597	2584

Jengdung Dantes makes a  $3 \times 7$  table:

1	1	2	3	5	8	13
21	34	55	89	144	233	377
610	987	1597	2584	4181	6765	10946

Jingdong Dantes' favorite column is column 2 (with values [1, 21, 377]), while Jengdung Dantes' favorite column is column 5 (with values [5, 144, 4181]). Thus, we have:

$$t_1 = 1 \cdot 5 = 5$$
  

$$t_2 = 21 \cdot 144 = 3024$$
  

$$t_3 = 377 \cdot 4181 = 1576237$$

Thus, the required output for the first test case is 5 + 3024 + 1576237 = 1579266. Modulo  $10^9 + 7$ , this is still 1579266.







# Problem K Bananagrams

### Time Limit: 3 seconds

Marion and Arion live in a magical place where bananas have letters on them. In particular, if the banana is c centimeters long, it will have exactly c letters on it. Thus, we can talk about the strings written in bananas. In particular, a banana of length c centimeters can be represented by a string of length c. For the purposes of this problem, assume that all bananas are integer length.

Marion has a big banana. Its length is n centimeters long and it contains the string s of length n. In order to be impressive, Arion wants to eat Marion's banana in one gulp. However, it is too long for Arion to eat. Thus, Arion decides to just eat a smaller banana, a banana of length k containing a string t of length k. However, this string t has to be a bananagram of the string s. Otherwise, Marion will not be impressed.

A bananagram of length k of a string s of length  $\ell$  is a string obtained by removing the first a letters of s and the last b letters of s where a and b are non-negative integers such that  $a+b=\ell-k$ , and then shuffling the remaining letters. Given the string s in Marion's banana, how many different bananagrams t can be in the smaller banana Arion decides to eat? Find the answer modulo  $10^9 + 7$ .

#### Input

The first line of input contains a single integer *t* denoting the number of test cases.

The first line of each test case contains two space-separated integers n and k. The second line contains a single string s of length n.

#### Output

Output a single line containing a single integer denoting the answer.

#### Constraints

 $1 \le t \le 6$   $1 \le k \le \ell \le 2 \cdot 10^5$ *s* consists of lowercase English letters.

Sample Input	Sample Output
1 3 bananaman	21







## Problem L Zoolander

### Time Limit: 2 seconds

Derek needs to go to save his friends who are in the gas station. His friends had a party where they would throw gas at each other. However, things went wrong when a stray cigarette suddenly lit the gas on fire. It's up to Derek to save them!<sup>1</sup>

The map is represented by an  $n \times m$  grid. Derek is at the cell marked with ^, >, v or <, and his friends are at the cell marked F. The rest of the grid is either . or #. Cells marked . are passable while cells marked # are obstacles.

Derek can navigate himself using the two actions listed below. He is able to do any of these two in exactly one second.

- Derek can move forward one space, as long as there is no obstacle in front of him.
- Derek can turn right and then move forward one space, as long as there is no obstacle in front of him when he attempts to move forward.

We clarify that Derek can't turn right if there is an obstacle directly to his right.

Moreover, it is a tragic illness that Derek cannot turn left. Unlike most people, he was not born an ambiturner.

What is the fastest time in which Derek can go and save his friends given the grid and the direction he is initially facing? If it is impossible, say so.

Input

The first line of input contains a single integer *t* denoting the number of test cases.

The first line of each test case contains two space-separated integers n and m. The next n lines each contains a string of length m denoting a row of the  $n \times m$  grid.

The direction that Derek is facing initially depends on the character used to represent his initial location: "north" for  $^$ , "east" for >, "south" for v, and "west" for <.

#### Output

For each test case, output a single line containing a single integer denoting the fastest time in which Derek can go and save his friends, or the string IMPOSSIBLE if it is impossible.

#### Constraints

$$\begin{split} &1 \leq t \leq 2 \cdot 10^5 \\ &1 \leq n, m \leq 100 \\ &\text{The sum of all } nm \text{ is } \leq 2 \cdot 10^6 \\ &\text{There is exactly one } ^{,} >, < \text{or v.} \end{split}$$



<sup>&</sup>lt;sup>1</sup>...after he gets his orange mocha frappuccino, of course!





There is exactly one F. The rest of the grid consists of either . or #.

Sample Input	Sample Output
2	10
5 6	IMPOSSIBLE
#	
.F	
^#	
·····	
5 4	
#F	
.>	

